QUANTUM COMPUTING IN A NUTSHELL

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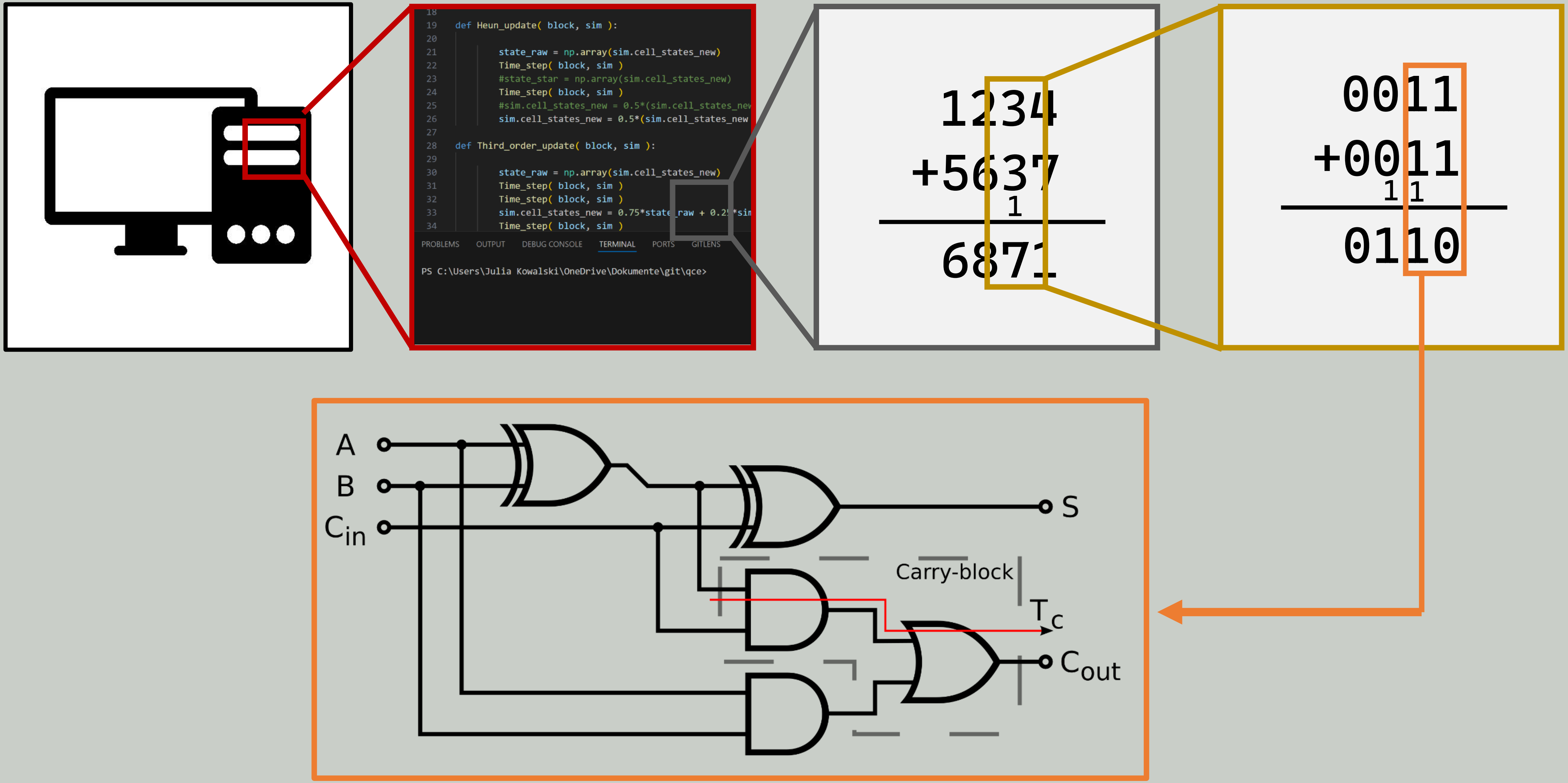
What happens in a (classical) computer?



Information is carried by X-BITS

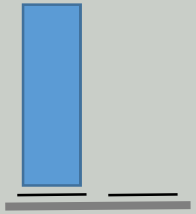
- A classical bit

$$B = \alpha\, 0 + \beta\, 1$$

$$\alpha, \beta \in \{0, 1\}, \quad \alpha + \beta = 1$$

admissibility condition

read out



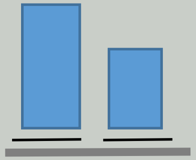
- A quantum bit

$$Q = \alpha\, |0\rangle + \beta\, |1\rangle$$

$$\alpha, \beta \in \mathbb{C}, \quad |\alpha^2| + |\beta^2| = 1$$

admissibility condition

measurement



Multiple X-BITS constitute a state

superposition

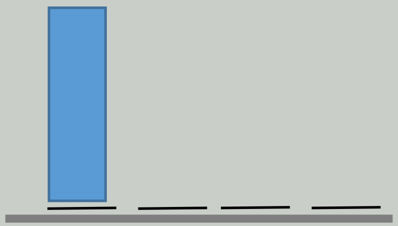
entanglement

- The joint state of two classical bits:

$$B_2 = (\alpha_1\, 0_1 + \beta_1\, 1_1)\,(\alpha_2\, 0_2 + \beta_2\, 1_2)$$

$$= \alpha_1\alpha_2\, 0_1 0_2 + \beta_1\alpha_2\, 1_1 0_2 + \alpha_1\beta_2\, 0_1 1_2 + \beta_1\beta_2\, 1_1 1_2$$

read out



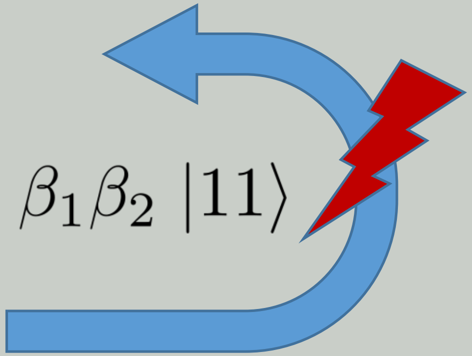
- The state of a two quantum bit system:

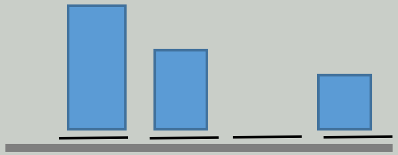
measurement

$$Q_2 = (\alpha_1\, |0\rangle + \beta_1\, |1\rangle)\,(\alpha_2\, |0\rangle + \beta_2\, |1\rangle)$$

$$= \alpha_1\alpha_2\, |00\rangle + \beta_1\alpha_2\, |10\rangle + \alpha_1\beta_2\, |01\rangle + \beta_1\beta_2\, |11\rangle$$

$$\tilde{Q}_2 = \gamma_1\, |00\rangle + \gamma_2\, |10\rangle + \gamma_3\, |01\rangle + \gamma_4\, |11\rangle$$





superposition

entanglement

reversibility

QBITs evolve under <u>unitary</u> transformations

- QBITs comply with Schrödinger's equation:

$$Q_{new} = U Q_{old} \quad \Rightarrow \quad Q_{old} = U^{\dagger} Q_{new}$$

<u>unitary</u> means $U U^{\dagger} = I$

Starting point:

- Feynman on December 29, 1959
  Annual American Physical Society meeting
  There is plenty of room at the bottom

- Feynman Lectures on Computation
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